Eur. J. Mech. B - Fluids 19 (2000) 453–457

S0997-7546(00)00116-3/FLA

On viscous forces on non-circular cylinders in low KC oscillatory flows

Bernard Molin *, Stéphane Etienne

Ecole Supérieure d'Ingénieurs de Marseille, 13451 Marseille cedex 20, France

(Received 10 October 1999; accepted 22 February 2000)

Abstract – This note provides a reanalysis of a result given by Bearman, Downie, Graham and Obasaju in 1985. It deals with viscous forces on fixed two-dimensional bodies in oscillatory flow, in the asymptotic case of low Keulegan–Carpenter number (KC) and high Stokes parameter (β). The flow is assumed to be laminar and attached (sharp corners are excluded). Bearman et al. show that, whatever the body shape, skin friction and pressure forces contribute equally to the viscous force, generalizing the result given earlier by Stokes in 1851 (see also Wang, 1968) for circular cylinders.

Here we show that their conclusion is ill-founded and that, presumably, it is only in the case of particular geometries that both components are equal.

viscous forces / non-circular cylinders / low KC oscillatory flows

1. Introduction

We start by following the same path as Bearman and his co-workers [1]: the non-viscous (potential) flow $\Phi^{(0)}$ is taken as a zero-order approximation, then boundary layer equations are used to obtain the correction to the tangential velocity (the outer pressure being taken as $-\rho\partial\Phi^{(0)}/\partial t$, in view of the small KC number). The first-order correction (in $\beta^{-1/2}$) to the normal velocity is obtained by integration of the continuity equation. This leads to a boundary value problem for $\Phi^{(1)}$, with a Neumann condition on the body, which need not be solved when one is only interested in the global loads, thanks to Green's second identity. All throughout, and unlike in the analysis of Bearman et al. [1], the problem is formulated by referring to the elementary potentials ψ_1 and ψ_2, associated with unit velocities of the body in x and y directions (the fluid being at rest at infinity). Skin friction and pressure loads are obtained through expressions that involve the gradients of ψ_1 and ψ_2 on the body, and that are different. A practical application is made for elliptic shapes, confirming that the two components are equal only in the case of a circular cylinder (see Stokes [2], Wang [3]).

2. Theory

The figure below illustrates the geometry. The outer fluid domain is infinite. Use is made of a curvilinear coordinate system (n, s), with the normal vector $\vec{n}$ pointing outward (*figure 1*).

We introduce the two elementary potentials ψ_1 and ψ_2 that verify the following boundary value problems

$$\Delta\psi_i = 0 \quad \text{in the fluid domain,} \tag{1}$$

$$\frac{\partial\psi_i}{\partial n} = n_i \quad \text{on the body contour,} \tag{2}$$

* Correspondence and reprints; e-mail: molin@esim.imt-mrs.fr

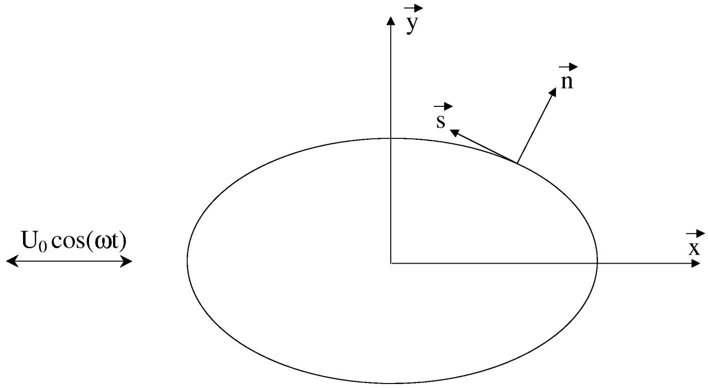

Figure 1. The coordinate system.

$$\nabla\psi_i \to 0 \quad R = \sqrt{x^2+y^2} \to \infty, \tag{3}$$

where (n_1, n_2) are the x and y components of the normal vector. ψ_1 and ψ_2 are the basic potentials from which such information as the added mass matrix (not relevant here) can be obtained: $m_{ij} = -\rho \oint \psi_j n_i \, \mathrm{d}s$.

The unperturbed incoming flow velocity being taken as $U_0 \cos\omega t$, in the x direction, the zero-order velocity potential is given by

$$\Phi^{(0)} = U_0(x - \psi_1)\cos\omega t. \tag{4}$$

The tangential velocity on the body is then

$$u^{(0)}(s,t) = -U_0\left(n_2 + \frac{\partial\psi_1}{\partial s}\right)\cos\omega t$$

or

$$u^{(0)}(s,t) = -U_0\left(\frac{\partial\psi_2}{\partial n} + \frac{\partial\psi_1}{\partial s}\right)\cos\omega t \tag{5}$$

from which the viscous additive correction in the boundary layer is obtained as

$$u^{(1)}(n,s,t) = U_0\left(\frac{\partial\psi_2}{\partial n} + \frac{\partial\psi_1}{\partial s}\right)\Re\left\{\mathrm{e}^{i\omega t-(1+i)\sqrt{\omega/(2\nu)}n}\right\} \tag{6}$$

in agreement with Bearman et al. [1].

They proceed by calculating the displacement thickness which, in our view, is not an appropriate concept when the outer velocity $u^{(0)}$ becomes nil occasionally (at $\omega t = (2n+1)\pi/2$). It is simpler to integrate the continuity equation, which can be simplified to $\partial u^{(1)}/\partial s + \partial v^{(1)}/\partial n = 0$, curvature playing no role at this stage. One obtains that the normal velocity $v^{(1)}$, at the edge of the boundary layer, is given by

$$v^{(1)}(s,t) = -U_0\left(\frac{\partial^2\psi_2}{\partial s\partial n} + \frac{\partial^2\psi_1}{\partial s^2}\right)\sqrt{\frac{\nu}{2\omega}}\Re\left\{(1-i)\mathrm{e}^{i\omega t}\right\}. \tag{7}$$

It ensues that the first-order correction to the outer potential velocity field obeys the Neumann boundary condition

$$\frac{\partial\varphi^{(1)}}{\partial n} = -U_0\left(\frac{\partial^2\psi_2}{\partial s\partial n} + \frac{\partial^2\psi_1}{\partial s^2}\right)\sqrt{\frac{\nu}{2\omega}}(1-i), \tag{8}$$

where $\Phi^{(1)} = \Re\{\varphi^{(1)} e^{i\omega t}\}$.

This also agrees with Bearman et al. [1] but they interpret (8) as a 'local source density', which is wrong, except in the particular case of a straight body. In the general case an integral equation must be solved to obtain $\varphi^{(1)}$.

However there is no need to do that when one is only interested in the global loads induced by $\Phi^{(1)}$. The dynamic complex pressure being

$$p = -i\omega\rho\varphi^{(1)}$$

the (complex) force component in the x direction is given by

$$f_{p1}^{(1)} = i\omega\rho \oint \varphi^{(1)} \frac{\partial\psi_1}{\partial n}\, \mathrm{d}s. \tag{9}$$

Green's second identity is used to transform this equation. Both $\varphi^{(1)}$ and ψ_1 decay sufficiently fast in the far field (as R^{-1}) that the integral over a remote contour vanishes. Therefore $f_{p1}^{(1)}$ is alternatively given by

$$f_{p1}^{(1)} = i\omega\rho \oint \frac{\partial\varphi^{(1)}}{\partial n} \psi_1\, \mathrm{d}s, \tag{10}$$

$$f_{p1}^{(1)} = -(1+i)\rho U_0 \sqrt{\frac{\omega\nu}{2}} \oint \left(\frac{\partial^2\psi_2}{\partial s \partial n} + \frac{\partial^2\psi_1}{\partial s^2} \right) \psi_1\, \mathrm{d}s \tag{11}$$

$$f_{p1}^{(1)} = (1+i)\rho U_0 \sqrt{\frac{\omega\nu}{2}} \oint \left(\frac{\partial\psi_2}{\partial n} + \frac{\partial\psi_1}{\partial s} \right) \frac{\partial\psi_1}{\partial s}\, \mathrm{d}s \tag{12}$$

through integration by parts.

From the expression (6) for the tangential velocity $u^{(1)}$ the skin friction component is obtained as

$$f_{f1}^{(1)} = -\rho\nu \oint \frac{\partial u^{(1)}}{\partial n} n_2\, \mathrm{d}s, \tag{13}$$

$$f_{f1}^{(1)} = (1+i)\rho U_0 \sqrt{\frac{\omega\nu}{2}} \oint \left(\frac{\partial\psi_2}{\partial n} + \frac{\partial\psi_1}{\partial s} \right) \frac{\partial\psi_2}{\partial n}\, \mathrm{d}s. \tag{14}$$

So the expressions obtained for the pressure force component and the skin friction component look very much alike. They only differ by the last terms in the integrands. But they yield different results, as can be seen from the case of elliptic shapes.

3. Results

We take the horizontal semi-axis a to be equal to one, and have the vertical (in y) semi-axis b vary from zero (flat plate) up to one (circle). The velocities $\partial\psi_i/\partial n, \partial\psi_i/\partial s$ are obtained from Milne-Thomson's [4] expressions (paragraph 6.33), duly removing the incoming flow velocity. On *figure 2* we show the values of

$$\frac{1}{L} \oint \left(\frac{\partial\psi_2}{\partial n} + \frac{\partial\psi_1}{\partial s} \right) \frac{\partial\psi_1}{\partial s}\, \mathrm{d}s \quad \text{and} \quad \frac{1}{L} \oint \left(\frac{\partial\psi_2}{\partial n} + \frac{\partial\psi_1}{\partial s} \right) \frac{\partial\psi_2}{\partial n}\, \mathrm{d}s,$$

that is, normalized by the perimeter L, the two integrals that appear respectively in the expressions for the pressure force (12) and skin friction (14). They are labelled as such on the figure. When $b/a = 0$ (flat plate) the

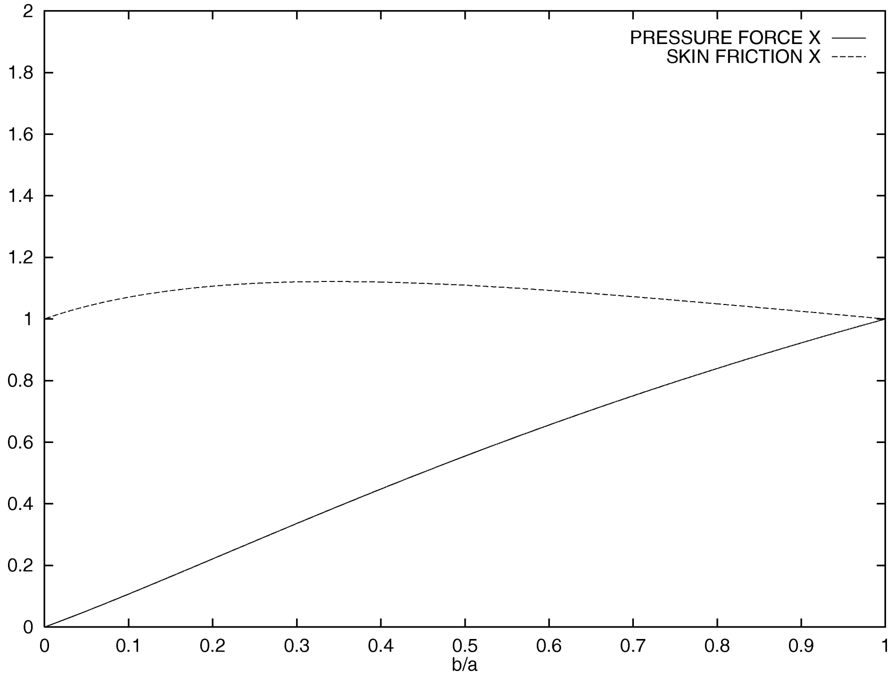

Figure 2. x-components of the pressure force and skin friction for an elliptic shape, versus b/a. Incoming flow in the x direction.

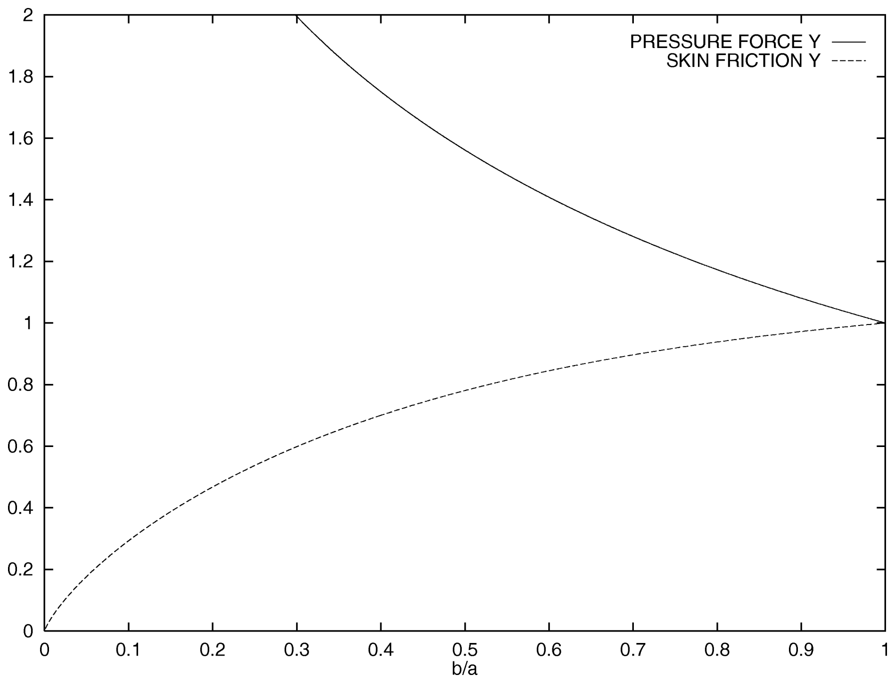

Figure 3. y-components of the pressure force and skin friction for an elliptic shape, versus b/a. Incoming flow in the y direction.

skin friction term is equal to 1, the pressure force term is nil. When $b/a = 1$ (circle) they are both equal to 1. Only in this case do they agree.

Figure 3 shows the corresponding results for an incoming flow in the y direction. The friction term drops to zero as the ellipse evolves from a circle back to a flat plate. On the other hand the pressure term blows up to infinity.

These results seem to be in better accordance with intuitive reasoning than those of Bearman et al. [1].

References

[1] Bearman P.W., Downie M.J., Graham J.M.R., Obasaju E.D., Forces on cylinders in viscous oscillatory flow at low Keulegan–Carpenter numbers, J. Fluid Mech. 154 (1985) 337–356.
[2] Stokes G.G., On the effect of the internal friction of fluids on the motion of pendulums, T. Camb. Phil. Soc. 9 (1851) 8–106.
[3] Wang C.-Y., On high-frequency oscillating viscous flows, J. Fluid Mech. 32 (1968) 55–68.
[4] Milne-Thomson L.M., Theoretical Hydrodynamics, Fifth Edition, Macmillan, New York, 1969.